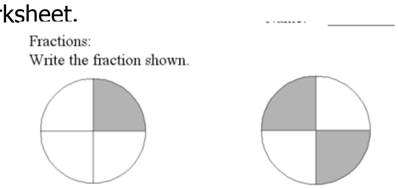
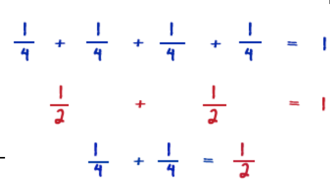
Levels of Learning Mathematics

The term "level" refers to the order that information presented mathematically is processed and learned. Mahesh C. Sharma, in "Learning Problems in Mathematics: Diagnostic and Remedial Perspectives" states that "almost all mathematics teaching activities, in most classrooms, take place at the abstract level. That is where most textbooks are; that is where most of the tests and examinations are." For students who have not mastered particular math content, he proposes the following order or "Levels of Math" as effective for teaching mathematics: intuitive, concrete/experiential, pictorial/representational, abstract, applications, and communication. The chart below explains each level and gives an example of what that level would look like in the classroom.

Levels of Learning	Explanation	Example
Intuitive	At the intuitive level, new material is connected to already existing knowledge. (The teacher checks that the connection is correct.) Introduce each new fact or concept as an extension of something the student already knows.	When a student is given three-dimensional circles cut into fractional pieces, he/she intuitively begin to arrange them into complete circles, thus seeing the wedges as part of a whole. 
Concrete/ Experiential	Manipulitives are used to introduce, practice and re-enforce rules, concepts, and ideas. Present every new fact or concept through a concrete model. Encourage students to continue exploring through asking other questions. 	Using the concrete model (in this case the wedges) helps the student learn the fractional names. As the student names the pieces, the instructors asks questions such as, "How many pieces are needed to complete the circle? Yes, four, so one out of these four is one fourth of the circle. As students continue to explore they may see that two of the quarters equal half the circle.
Pictorial/ Representational	Picture, diagram, image is used to solve a problem or prove a theorem. Sketch or illustrate a model of the new math fact. Pictorial models are those pictures often provided in textbook worksheets.	When the student has experienced how some pieces actually fit into the whole, present the relationship in a pictorial model, such as a worksheet. 
Abstract	Student is able to process symbols and formulae. Show students the new fact in symbolic (numerical) form. 	After the student has the concrete and pictorial models to relate to, he can understand that 1/4 + 1/4 is not 2/8. Until this concept has been developed, the written fraction is meaningless to the student.

Applications	Student is able to apply a previously learned concept to another topic. Ask student to apply the concept to a real-life situation. The student can now approach fractions with an understanding that each fraction is a particular part of a whole. The instructor can now introduce word problems without illustrations because students have images in their heads.	A student who is asked to give a real-life example or situation might respond with 1/4 cup of flour + 1/4 cup of flour equals 1/2 cup of flour. 
Communication	The student is able to convey knowledge to another student reflecting an embedded understanding and the highest level of learning. The student's success in this task reflects an embedded understanding and the highest level of learning.	Ask students to convey their knowledge to other students, i.e., students must translate their understanding into their own words to express what they know.

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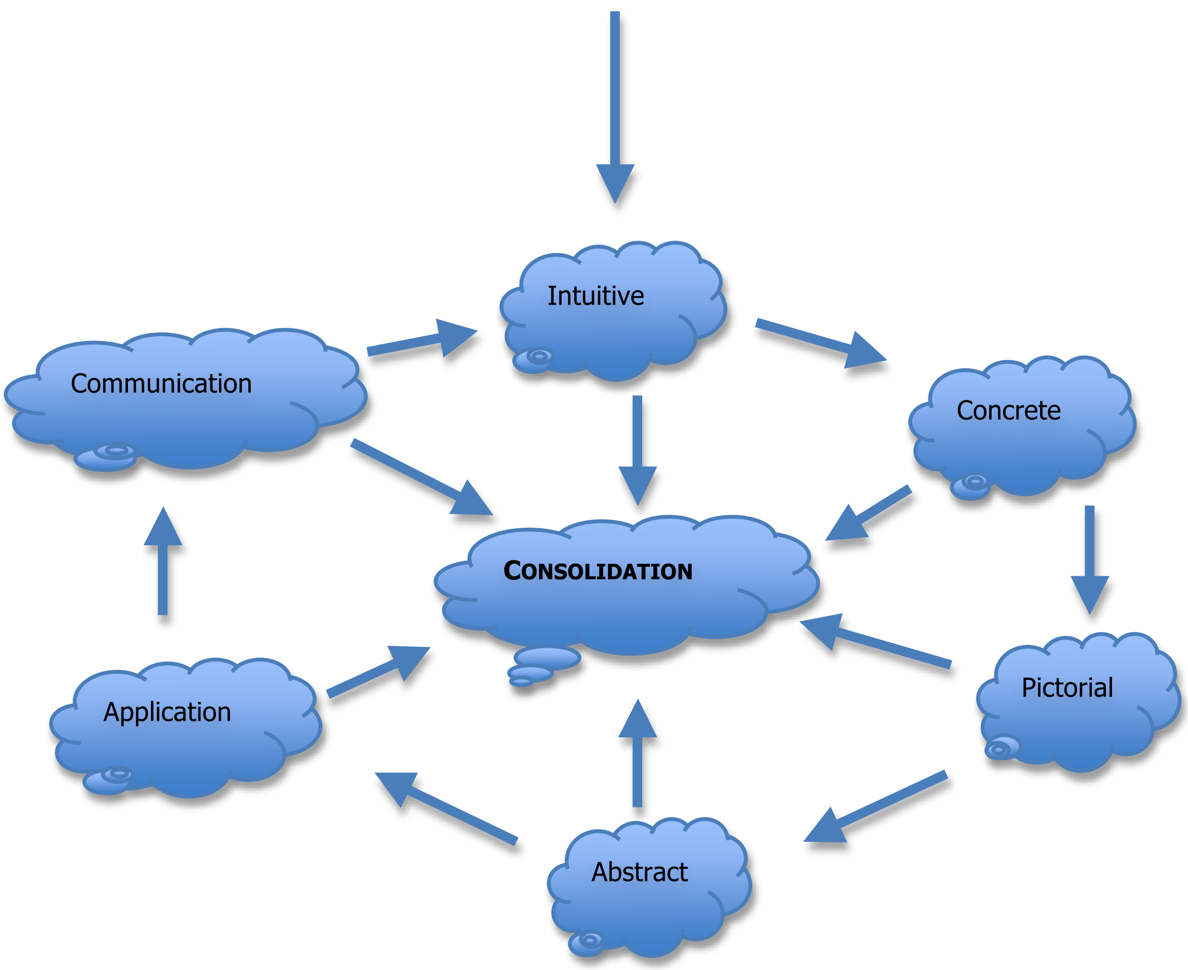

Sharma wrote, "The mastery of a given mathematical concept passes from the intuitive level of understanding to the level where the student can explain how he has arrived at a particular result and can explain the intricacies and the concept. In many of the regular classroom teaching situation, the teach…may begin at the abstract form of the concept. As a result the student may face difficulty in learning the concept or procedure being taught. Even if he has understood the procedure for solving that problem he may soon forget it. Later when the teacher begin a new concept he may assume, incorrectly, that the mastery in the previous concept is still present and therefore may begin the new concept at a higher level, i.e., the abstract level, creating difficulty for the student. This cycle continues and eventually the student begins to lose the teacher's explanations. The student begins to have difficulty in learning mathematics, which then results in the failure and that develops a fear of mathematics."

This hierarchy of learning can in turn offer a structure for the teacher to follow. If our goal is for students to learn well and do well on test, instructors should do the following when planning instruction in mathematics:

- Introduce concepts at the intuitive level, and lead students through all the levels to the communication level.
- Make sure that the student understands the linguistic, conceptual, and procedural components of the concept.
- Over-teach the concept, i.e., repeatedly use it in one form or another.
- Take he student to a higher level than is required on test (the abstract), i.e., take the student at least to the application level.

Adapted from:

Massachusetts Adult Basic Education Curriculum Framework, Massachusetts Department of Education, Adult and Community Learning Services, October 2005.

Mahesh C. Sharma, Handout entitled, "Learning Problems in Mathematics: Diagnostic and Remedial Perspectives."